Tutorial problems (23750) for “Solar Energy“ lecture (23745), WS 2014/2015

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Tutorial Questions #1:

1. **Solar Radiation**

a) Understanding the Black Body radiation

Planck’s law of radiation describes the photon distribution of a black body. A black body is characterized by total absorption of light $\alpha(\hbar\omega) = 1$. That means that one can assume that a black body is totally occupied with photons. To obtain the density of photons $n_\gamma(\hbar\omega)$ inside the black body it is needed to know the total number of states $D_\gamma(\hbar\omega)$, which can be occupied and the distribution of the particles according to their energy $f_\gamma(\hbar\omega)$ over those states. As a result we get:

$$n_\gamma(\epsilon = \hbar\omega) = \int_0^\epsilon D_\gamma(\hbar\omega) f_\gamma(\hbar\omega) d(\hbar\omega)$$

or equivalently:

$$dn_\gamma(\hbar\omega) = D_\gamma(\hbar\omega) f_\gamma(\hbar\omega) d(\hbar\omega)$$

The density of states is:

$$D_\gamma(\hbar\omega) = \frac{\partial^2 N_\gamma}{\partial V \partial(\hbar\omega)} = 2 \times \frac{(4\pi/3)p^3}{(h^3/V)},$$

where N_γ is the number of states. **Try to derive this expression by assuming that the states are restricted to a sphere in the momentum space and the distance between the states is given by Heisenberg’s uncertainty relation $\boldsymbol{\Delta x \Delta p \geq h}$. Where does the factor of two come from?** The distribution function of photons can be found in the literature. Verify that the following expression for the photon density is true:

$$\frac{dn_\gamma}{d(\hbar\omega)} = \frac{D_\gamma(\hbar\omega)}{4\pi} f_\gamma(\hbar\omega) d\Omega = \frac{(\hbar\omega)^2}{4\pi^3\hbar^3c^3} \frac{1}{\exp(\hbar\omega/k_BT) - 1} d\Omega$$

where Ω is the solid angle.

b) Radiation from the sun towards the earth

Calculate the radiation power from the sun. What is the amount of energy which the earth receives during one whole day? **Try** to derive the equations from the equation for the black body radiation. What is the solar constant? How is the air mass affecting those values?

c) Modeling a real solar cell, Part 1

Imagine a solar panel on top of a KIT building. Calculate the incident radiation on the panel on the 21st of March. How much the panel must be tilted with respect to the horizontal to receive an incident radiation power of 750 W/m^2?

2. **Carrier dynamics in Semiconductors**
   a) **Diffusion current density**
   Derive the current density for charge carriers inside a bulk material. Use as a starting point the following expression for the particle flux:

$$j = q \cdot \phi = \frac{l}{2\tau}\big(n(x) - n(x+l)\big),$$

with q as the charge, l the mean free path and $\bar{t}$ the time between two collisions. Assume that the mean free path is small compared to the system size. The result should be:

$$j_n = qD_n \frac{\partial n(x)}{\partial x}, j_p = -qD_p \frac{\partial p(x)}{\partial x}$$

with $D = \frac{l^2}{2\tau}$ as the diffusion constant.

   b) **Field induced current density (Drift current)**
   Beside currents which are due to a density gradient there are also field induced currents. A popular model for describing charge carriers inside a semiconductor is the Drude model:

$$m\dot{v} + \frac{m}{\tau} v_D = -qE,$$

where m is the mass of the charge carrier, q the charge, E an electrical field (external or internal caused by doping) and v_D the so called drift velocity. Assume the stationary case and derive an expression for the current density. Use the relation between conductivity σ and mobility μ:

$$\sigma = qn\mu$$

to find the following expressions:

$$j_n = qn\mu_n E, j_p = -qp\mu_p E$$

   c) **Diffusion equation**
   Use the continuity equation for the density (not charge density) to obtain the Diffusion equation. Assume that the electrical field is spatial homogenous:

$$\frac{\partial n}{\partial t} = -(D_n + \mu E)\frac{\partial n}{\partial x}$$

How can one model the recombination and generation of charge carriers?

3. **Semiconductors**

   a) **What is a Semiconductor?**

   Differentiate qualitatively between a metal and an insulator. Which role plays the Fermi energy? What can be stated for the electron distribution at $T = 0\ K$?

   b) **Bandstructure**

   What are electronic bands? Do bands exist in disordered media? Where does the band gap come from?

   c) **Diffusion potential**

   Use the derived expressions for the currents in **2. Carrier dynamics in Semiconductors** to find an expression for the Diffusion potential U_D. Assume a pn-junction with the following doping concentrations $n = 10^{19}/cm^3$ and $p = 10^{16}/cm^3$. Use the Einstein relation $D/\mu = kT/e$.